Negative Integers

Lecture 7
Section 2.5

Robb T. Koether

Hampden-Sydney College

Fri, Jan 24, 2014

1. Fixed-Length Arithmetic
2. Two's Complement
3. A Binary Subtractor
4. Assignment

Outline

1. Fixed-Length Arithmetic
2. Two's Complement
3. A Binary Subtractor
4. Assignment

Fixed-Length Addition

- Represent 150 and 106 as 8-bit integers.
  - $150 = 10010110$.
  - $106 = 01101010$.
- Express the sum as an 8-bit integer.
  - $10010110 + 01101010 = 00000000$.
  - Carry-out bit is thrown away.
- Conclusions
  - $150 + 106 = 0$.
  - $150 = -106$.

Example

Example (`UnsignedInt.cpp`)

- Run `UnsignedInt.cpp`.

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Two's Complement

- For binary numbers of fixed length n, the two's complement of a number a is
$$2^n - a.$$
- For any integer a, the integer $-a$ is stored as the two's complement of a.
- The two's complement of the two's complement of a is a, just like the negative of the negative of a is a.

Two's Complement

- To find the two's complement of an n-bit binary number:
  - Reverse each bit, including leading zeros.
  - Add 1 to the result.
- Reversing each bit is equivalent to subtracting from $111\ldots1 = 2^n - 1$.

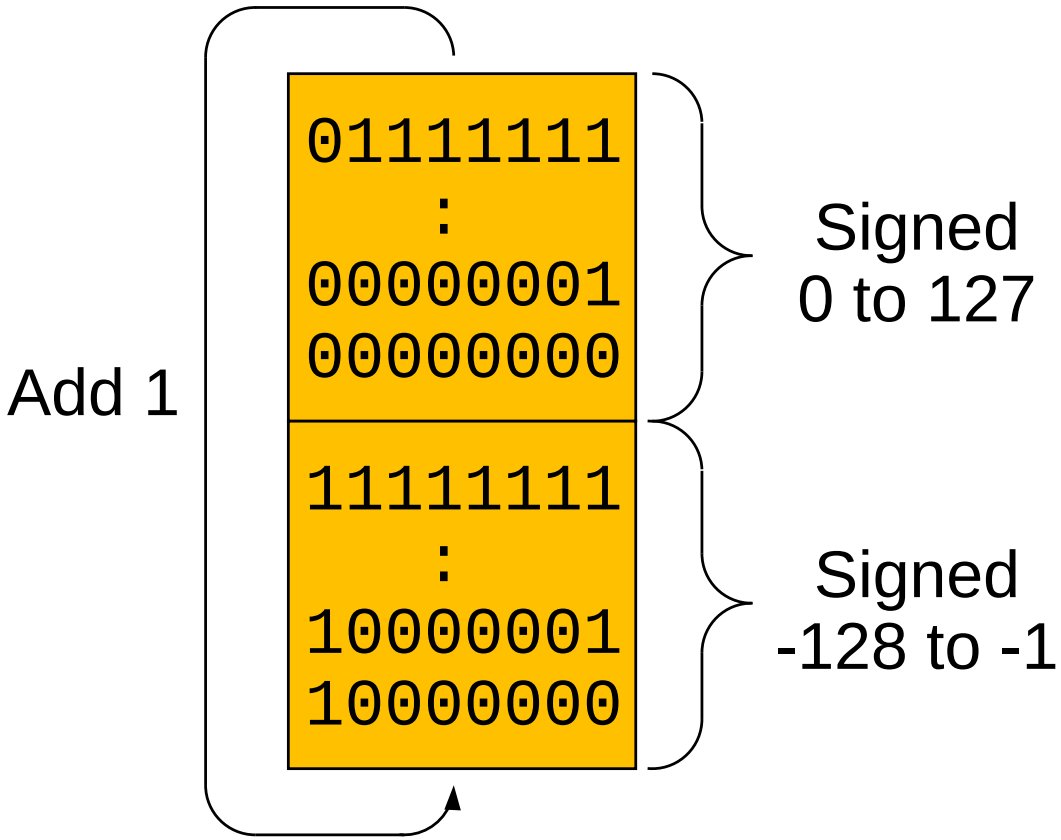
Two's Complement

- If we store 10010110, how can we tell whether it represents 150 or -106?
- If we store 01101010, how can we tell whether it represents 106 or -150?

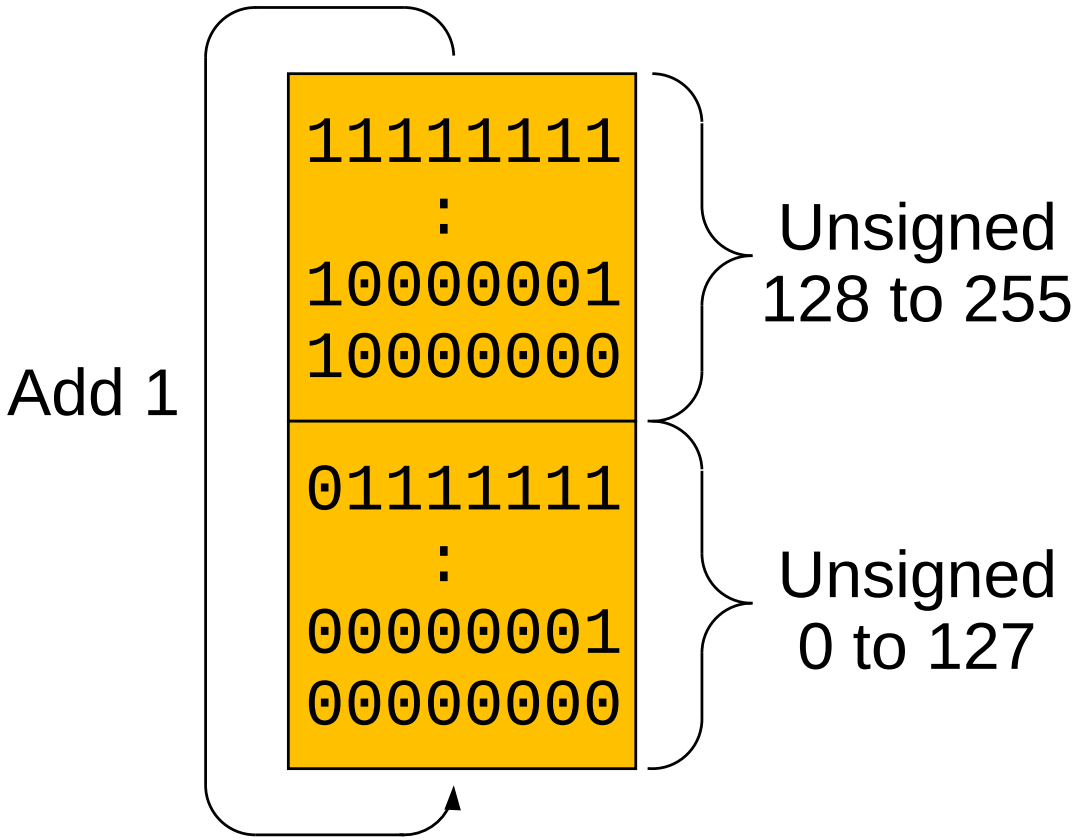
Signed vs. Unsigned

- For signed integers,
  - If the high-order bit is 0, the integer is positive (from 0 to $2^{n-1} - 1$).
  - If the high-order bit is 1, the integer is negative (from -2^{n-1} to -1).
- For unsigned integers,
  - If the high-order bit is 0, the integer is from 0 to $2^{n-1} - 1$.
  - If the high-order bit is 1, the integer is from 2^{n-1} to $2^n - 1$.

Signed Integers



Unsigned Integers



Signed vs. Unsigned

Stored Bits	Signed Value	Unsigned Value
00000000		
00000001		
01111111		
10000000		
10000001		
11111111		
01000000		
11000000		

- Fill in the values.

Example

Mixing Types

```
short s = -1;
int i = s;
unsigned int j = s;
```

- In a C program, what happens when we execute the code above?

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A Binary Subtractor

- A binary subtractor may be created from a binary adder by
  - Inverting the second operand, and
  - Adding 1 by setting the initial carry-in to 1.
- Design a binary subtractor in Logisim.

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- Read Section 2.5, pages 84 - 94.
- Exercises 23, 24, 27, 28, 31, 33, 35, 36, 37, 39, 42, page 94.